The real demand-side theory

General Equilibrium and Disequilibrium
in the Circular Flow Model

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Abstract

The present paper examines the dynamics of the market economy in the framework of a circular flow model. This model is different from the conventional input-output models in that it uses not only the technical but also the distribution coefficients. With these coefficients it is possible to analyze not only structural relations but also general equilibrium. The most important result from our analysis is the fact that disequilibrium is a state of the free market economy that is internally consistent and independent. As such, it is neither equilibrium not yet reached nor equilibirum disturbed, as in neo-classical models (by Walras, Pareto, etc.) and has nothing to do with money disappearing (via hoarding or liquidity preference), as in Keynes theory. The paper attempts to analyze growth. Saving and investment are the driving forces of growth, but they also cause lack of demand and lead into economic crashes, such that the present equilibrium analysis also provides the basis for a new theory of cyclical fluctuations (economic cycles).

KEYWORDS: Equilibrium and disequilibrium, distribution coefficients, prices, productivity, growth, general savings equation, economic cycles

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Thrift makes possible a high rate of accumulation and yet sets obstacles in the way of achieving it. This paradoxical operation of the capitalist rules of the game is one of the main subjects which we hope to be able to elucidate by economic analyses.

Joan Robinson

The originality of mathematics consists in the fact that in mathematical science connections between things are exhibited which, apart from the agency of human reason, are extremely unobvious.

Alfred N. Whitehead

Introduction

The circular flow model of the economy implies several analytical concepts that are entirely different from those of the particle-mechanic model. This paper develops a couple of these concepts. They correspond most closely to the method of nodes, also called the matrix method, by Hans Peter.[1] Peter was the

[1] *Mathematische Strukturlehre des Wirtschaftskreislaufes*, pp. 15 and 82.

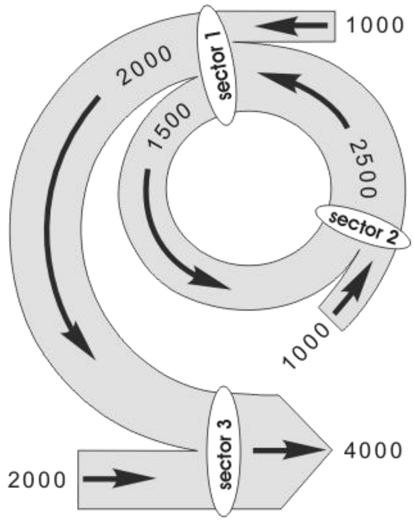
first economist to derive the general theory on the structure of a circular flow model.

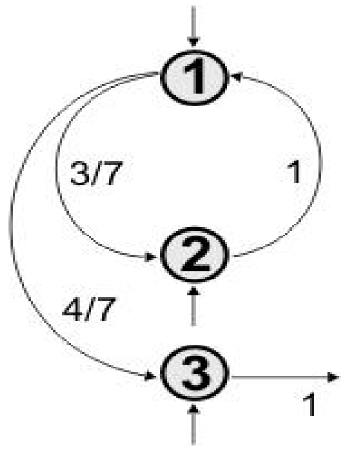
In his circular flow model single firms or economic sectors are generally called *poles*. What is transformed between two poles Peter calls stream. A stream flows in a particular direction, and its value is called *stream width.* The pole width is a magnitude that indicates the quantity of economic value that flows through a pole in a specific time period. To close the circular flow, the condition must hold that the sum of the widths of those circular streams that enter into the pole, is equal to the sum of the widths of the outgoing streams. The inflowing circular streams or *inputs* to a pole j can, in general mathematical notation, be called $x_{1j}, x_{2j}, x_{3j}, \dots, x_{kj}, \dots, x_{nj}$ and the outgoing streams or *outputs* $x_{j1}, x_{j2}, x_{j3}, \dots, x_{jk}, \dots, x_{jn}$. The entire width of pole j is denoted by x_j.With the help of the circular streams, structural coefficients can be created and defined as "proportion of stream width to pole width". There are two sets of such coefficients, according to whether you look at the relation of stream width to the width of the pole into which it (the stream) flows in, or to the one from which it flows out. The coefficients of the first type are customarily called *technical coefficients,* and those of the second type we will call *distribution coefficients:*

$$\tau_{kj} = x_{kj} / x_j \qquad \text{and} \qquad \delta_{jk} = x_{jk} / x_j \ .$$

The *technical coefficients* represent the proportions in which input factors have to be combined for production in a specific sector. They are, however, a gross simplification of reality as factor proportions are not rigid. Even so they are well suited for the representation of economic systems - especially the *static ones* - as they make possible the representation of complex and multi-level structures, and therefore real economies, using empirical data (as in Leontief tables). The re-switching of techniques is among the most important insights that we owe to the analysis of production with technical coefficients.

The *distribution coefficients* do not relate the characteristics of a single sector, they relate to the structure of the overall economic system. The following example shows how distribution coefficients are derived from numerical data. We will use the following numerical example with the picture (flowchart) later to illustrate the mathematical relationships and conclusions in a simpler way. However, only the distributive coefficients of sector 1 will change there. Now the value 3/7 or 0.429 of the coefficient δ_{12} results from the ratio of the outgoing substream 2000 to the total output 3500 of the sector 1, the value 4/7 or 0.571 of the coefficient δ_{13} from the ratio of the substream 1500 to the same total output. As the distribution coefficients do not reflect state of technology the technological status of single sectors, as is the case with technical coefficients, they are not bound to the assumptions of constant factor proportions (returns to scale), and with that they are not confined to the representation of linear production changes (proportional dynamics). This makes a kind of macroeconomics analysis possible that includes technical change.





It is our main task to find out which equilibrium conditions product streams and their market prices have to satisfy to maintain general equilibrium, and when they are not satisfied such that there is a lack of demand.

Let's start out with the general statement that the market value of overall production of any economic sector j, is determined by two kinds of production costs: by the cost of all used up technological inputs (raw materials, intermediate goods and machines), and by expenditure on various kinds of services absorbed by the agents that are paid as net income. The sectors' net income includes salaries, interest, profit, etc., but for our analysis only overall net income is relevant, its distribution within the sector is irrelevant. It is enough to assume that such income exists in the first place, i.e. each sector has a surplus to be distributed, which we will call $\hat{y}$. The cost structure of the overall production in each sector j in an economy with n sectors can be represented in an algebraic equation as follows:

$$x_{1j}\,p_1 + x_{2j}\,p_2 + x_{3j}\,p_3 + \ldots + x_{nj}\,p_n + \hat{y}_j = x_j\,p_j\,.$$

By x_{1j} we mean that physical quantity of producer goods, which is carried from sector 1 to sector j; with x_{2j}, analogously, the quantity of producer goods delivered by sector 2 to sector j, … etc. If sector j purchases these goods at prices p_1, p_2, … per physical unit, it produces a physical quantity x_j of goods, or a real overall output whose nominal value at a nominal price p_j yields a gross income of $x_j\, p_j$ which we describe with the symbol y_j. Part of this gross income - the expenditure on various services - goes to net income. If the first term of the equation is divided by x_1 and at the same time multiplied with it, it can be represented as the product of two multipliers x_{1j}/x_1 und $x_1 \cdot p_1$. The first multiplier is by definition a distribution coefficient, specifically δ_{1j}; the second multiplier is gross income of sector 1, which we call y_1. If in the second term, the same is performed with the variable x_2, and in the same way in the other terms, the original equation can be written as:

$$\delta_{1j}\, y_1 \; + \; \delta_{2j}\, y_2 \; + \; \delta_{3j}\, y_3 \; + \; ... \; + \; \delta_{nj}\, y_n \; + \; \hat{y}_j \; = \; y_j \, .$$

If an economy has n sectors, we obtain a system of n equations, which, in analogy to the previous equation, can be written in matrix form as follows:

$$\mathbf{\Delta}_{nn}\, \mathbf{y}_n \; + \; \hat{\mathbf{y}}_n \; = \; \mathbf{y}_n \, . \qquad \text{(a)}$$

The distribution coefficients δ_{kj} in this system of equations form a quadratic (two-dimensional) matrix, and its variables form (one-dimensional) vectors. It is sufficient to assume that this income exists in the first place, i.e. that each sector has more value than the required minimum for exchange and that therefore a *surplus* to be distributed exists. The matrix $\mathbf{\Delta}_{nn}$ has certain properties, which we will use. If sectors *1* to *h* are those that make producer goods, and sectors *h+1* to *n* make consumer goods, then matrix $\mathbf{\Delta}_{nn}$ has a form as shown below. This matrix can, by a procedure of decomposition, be taken apart into simpler matrices, as shown to the right.

$$\left[\begin{array}{cccc:cccc} \delta_{11} & \delta_{21} & ... & \delta_{h1} & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ \delta_{1h} & \delta_{2h} & ... & \delta_{hh} & 0 & 0 & ... & 0 \\ \hdashline \delta_{1\,h+1} & \delta_{2\,h+1} & ... & \delta_{h\,h+1} & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ \delta_{1n} & \delta_{2n} & ... & \delta_{hn} & 0 & 0 & ... & 0 \end{array}\right] \longrightarrow \left[\begin{array}{c:c} \mathbf{\Delta}_K & \mathbf{0} \\ \hdashline \mathbf{\Delta}_C & \mathbf{0} \end{array}\right]$$

The distribution coefficients whose first index number is *h+1* or bigger, have value 0, as the producers of consumer goods do not turn any goods back into the economy. This makes it possible to reduce matrix $\mathbf{\Delta}_{nn}$ to two smaller matrices, $\mathbf{\Delta}_{K}$ and $\mathbf{\Delta}_{c}$. The notation of the sub-matrices requires some explanation. The first matrix $\mathbf{\Delta}_{K}$, indexed by K, captures those sectors that produce producer goods, and the matrix $\mathbf{\Delta}_{c}$ the others that produce consumer goods. It would be mathematically correct to add indexes to both matrices that indicate the number of rows and columns. We will omit those indexes, however, so that the matrix equations become more similar to regular equations and can be treated as such, and so that the explanations below can be followed more easily.

The two vectors of equation system (a) can be broken down vertically, by a method of decomposition, into two vectors each: Vector $\mathbf{y}_{n}$ in $\mathbf{y}_{K}$ and $\mathbf{y}_{c}$ and vector $\hat{\mathbf{y}}_{n}$ analogously in $\hat{\mathbf{y}}_{K}$ and $\hat{\mathbf{y}}_{c}$. Again, the subscripts provide information on whether the vector describes sectors that provide producer goods (K), or those that provide consumer goods (C). We will not continue using the subscripts that describe the mathematical dimension. From the "top" and the "bottom" part of equation system (a), two equation systems can be formed:

$$\begin{aligned} \mathbf{\Delta}_{K}\,\mathbf{y}_{K} + \hat{\mathbf{y}}_{K} &= \mathbf{y}_{K} \\ \mathbf{\Delta}_{C}\,\mathbf{y}_{K} + \hat{\mathbf{y}}_{C} &= \mathbf{y}_{C}\,. \end{aligned} \tag{b}$$

As this is a system with *surplus*, similar to Piero Sraffa (*Production of Commodities by Means of Commodities*), the following equation holds:[2]

$$\underbrace{\mathbf{1}\,(\mathbf{I} - \mathbf{\Delta}_{K})\,\mathbf{y}_{K}}_{\text{Term 1}} + \underbrace{\mathbf{1}\,\hat{\mathbf{y}}_{c}}_{\text{Term 2}} = \underbrace{\mathbf{1}\,\mathbf{\Delta}_{c}\,\mathbf{y}_{K}}_{\text{Term 3}} + \underbrace{\mathbf{1}\,\hat{\mathbf{y}}_{c}}_{\text{Term 4}}\,. \tag{c}$$

Equation (c) describes the *equilibrium condition* in the consumer goods market. As it is tacitly assumed that the market for producer goods has cleared, this describes equilibrium in the overall economy. The reproduction period that this equation refers to, will in the following be described with index t. However, some components of equation (c) are not formed in reproduction period t but in the prior t-1, such that its price has been fixed in the previous period. To

[2] We add first the left and then the right side of the above system of equations (b), using a summation vector ($\mathbf{1}$), which yields

$$\mathbf{1}\,\mathbf{\Delta}_{K}\,\mathbf{y}_{K} + \mathbf{1}\,\hat{\mathbf{y}}_{K} + \mathbf{1}\,\mathbf{\Delta}_{c}\,\mathbf{y}_{K} + \mathbf{1}\,\hat{\mathbf{y}}_{c} = \mathbf{1}\,\mathbf{y}_{K} + \mathbf{1}\,\mathbf{y}_{c}\,.$$

If we put the terms into a different order now and use the diagonal *unit matrix* ($\mathbf{I}$) we receive

$$\mathbf{1}\,(\mathbf{I} - \mathbf{\Delta}_{K})\,\mathbf{y}_{K} + (\mathbf{1}\,\mathbf{y}_{c} - \mathbf{1}\,\hat{\mathbf{y}}_{K}) = \mathbf{1}\,\mathbf{\Delta}_{c}\,\mathbf{y}_{K} + \mathbf{1}\,\hat{\mathbf{y}}_{c}\,.$$

Since the value of all consumer goods ($\mathbf{1}\,\mathbf{y}_{c}$) in static equilibrium equals the sum of all net income ($\mathbf{1}\,\hat{\mathbf{y}}_{c} + \mathbf{1}\,\hat{\mathbf{y}}_{K}$), the expression in the second parenthesis has the value $\mathbf{1}\,\hat{\mathbf{y}}_{c}$, from which direct follows equation (c).

take this temporality into account, we now examine the meaning of the terms of equation (c) more closely:

Term 1: It describes the entire net income of those sectors that produce producer goods. It is the income that *effectively* remains in these sectors, when output is completely sold, and the technological goods for the following production period are bought. Even though the values of $\mathbf{\Delta}_K$ and $\mathbf{y}_K$ are mainly determined at the moment of exchange at the end of the production period, it is still correct to denote them with index t, as in equation (d).

Term 2: It describes the sum of net income of those sectors that produce consumer goods. All this income is formed in the course of production period t, which is why we denote it also with subscript t.

Term 1 and *Term 2* together represent *effective demand* in the market for consumer goods.

Term 3: Its components are the cost that arises from the production of producer goods. These come from used and depreciated producer goods that the sectors had bought at the beginning of production, at the prices going then, which is why they carry the index $t-1$. These costs are part of supply, and when they are realized, they go into an amortization fund.

Term 4: It represents net income of those sectors that produce consumer goods, just like term 2. This time they are to be understood as the production cost included in consumer goods, i.e. they are now part of supply.

Terms 3 and 4 make up the *effective supply* on the market for consumer goods.

If we now add time subscripts to the variables in equation (c), we obtain a time-indexed equilibrium condition

$$\underbrace{\mathbf{1}\,(\mathbf{I} - \mathbf{\Delta}_K^{t})\,\mathbf{y}_K^{t} + \mathbf{1}\,\hat{\mathbf{y}}_c^{t}}_{\textit{Effective Demand}} = \underbrace{\mathbf{1}\,\mathbf{\Delta}_c^{t-1}\,\mathbf{y}_K^{t-1} + \mathbf{1}\,\hat{\mathbf{y}}_c^{t}}_{\textit{Effective Supply}} . \tag{d}$$

Matrix $\mathbf{\Delta}_c^{t-1}$ and vector $\mathbf{y}_K^{t-1}$ connect reproduction period t to the past, such that our analysis provides a snapshot from the continuing economic process. The reproduction periods are not simply arranged along a time axis and pushed together (like in a string of pearls), as in comparative statics, but they overlap, like links in a chain, such that each reproduction period follows from the previous one both structurally and functionally.

Equation (d) is now the basis for our analysis of the *dynamic characteristics of equilibrium and disequilibrium.* We would like to analyze economic growth, as growth is the most important case within dynamic economic analysis. However, this is not about growth per se but about the factors and variables in growth that are *relevant for equilibrium.* We need to find out which they are.

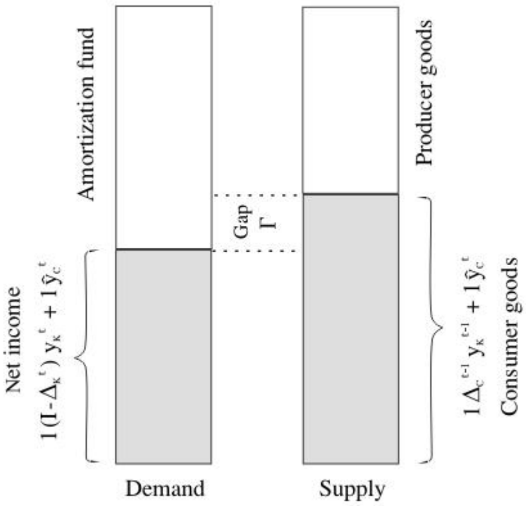
For *one* variable it can clearly be stated, by just simply looking at it, that it is *not* among those relevant for equilibrium - the (nominal) prices of consumer goods, determined by the vector $\hat{\mathbf{y}}_C^t$. This vector appears as a term on both sides of equation (d), such that its effect is nullified. General equilibrium cannot be depending on the nominal consumer prices. Applied to some specific conditions, this means that in an economy that produces and exchanges only consumer goods, there cannot be general disequilibrium. Incidentally, this is a theoretical conclusion explaining the historical fact that in pre-capitalist economies where (almost) all suppliers were producers of consumer goods, and consumer goods markets dominated, the problem of a lack of demand and general over-production did not exist.

1 Transition from a static economy to the growth path

If an economy wants to grow, it needs for its activity on the *higher* level, larger amounts of investment, i.e. of producer goods. At the beginning of the growth period, these are not available in a static system. They have to be produced first. One of the possibilities to do this is the *reallocation of resources.* This means making available a larger amount of producer goods than before to the producers of input goods (raw materials, intermediate products and machines), by temporarily allocating fewer to the producers of consumer goods. Marx was the first who investigated this reallocation of resources in the second volume of his *Kapital* - the so-called extended reproduction. He based himself entirely on numerical examples, without using a systematic and logically stringent method, and therefore he missed the equilibrium and lack of demand problem. The problem was also missed in the “Cambridge debate” on capital theory based on the model by Sraffa, which by no means lacks mathematical stringency and consistency. Sraffa’s circular flow model uses technical coefficients, however it is not possible to make the problem of equilibrium and lack of demand visible with the technical coefficients. The representation of the problem of lack of demand requires the use of the *distribution coefficients.*

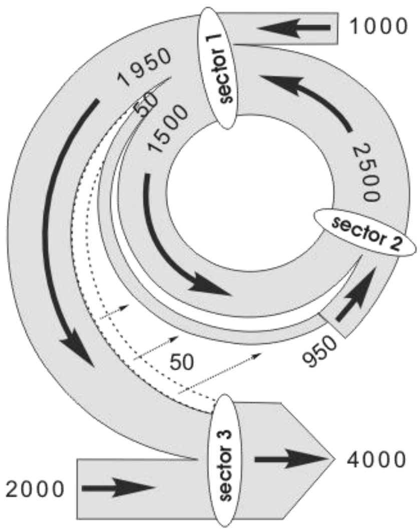
When the economy starts to grow after the reallocation of producer goods, some δ-coefficients of the matrix $\mathbf{\Delta}_K^t$ in equation (d) must become bigger. Since reallocation does not change anything in vector $\mathbf{y}_K^t$, term **1** in equation (d) becomes smaller. Reallocation does not change anything on the right-hand side of the equation, such that *effective demand* (the left-hand side) becomes smaller than *effective supply* (the right-hand side). The resulting disequilibrium is illustrated in a graph to be completed shortly. With it we are pointing out that the previous equation (d) does not capture total demand and total supply. Demand also includes income from amortization, and supply also includes total producer goods produced. In the graph below these two variables, or rather, aggregates, are taken into account also.

Graph 1:



The graph also highlights that in our analysis the sum of all costs is equivalent to the sum of all income. We have never questioned this identity for the economy as a whole. Our last equality (d), or rather, inequality, only refers to the lower (grey) parts on the left and the right bars. This equation continues to be the basis of our equilibrium or rather disequilibrium analysis. What is not comprised in this equation (d) is not relevant for our conclusions, as far as equilibrium or disequilibrium is concerned. As the (grey) parts above are not equal, a case of disequilibrium is to be expected.

A numerical example better helps us to understand how the equation (d) becomes an inequation. For this we already have the above picture (flow chart) at our disposal, which shows a simple economy in equilibrium. But now sector 2 is to make investments and is therefore buying more goods (+ 50) from sector 1 as usual. This is of course only possible by reallocation of the real goods, that means if at the same time sector 3 is buying less goods (- 50) than before. This gives the distributive coefficient δ_{12} a higher value (0.443) than before (0.429), δ_{13} is correspondingly smaller (0.557) than before (0.571).



Left part of equality (**d**):

Sector 1:	1000
Sector 2:	950
Sector 3:	2000
==================	
Net income: =	**3950**

Right part of equality (**d**):

Sector 1:	0
Sector 2:	0
Sector 3:	4000
====================	
Consumer goods: =	**4000**

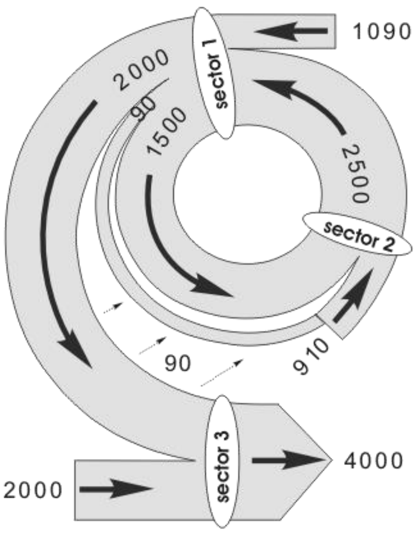
The picture shows that, after reallocation, the available net income is no longer sufficient for the purchase of all final consumer goods. Even if *all* of the available net income were spent on consumer goods, a certain amount Γ of final consumer goods unsold would still remain. There is disequilibrium. It is true that these unsold non-marketable consumer goods correspond to amortization, i.e. the cost of the producer goods used and depreciated during their production. These can be used for the purchase of the overhang in consumer goods, such that, in principle, the economy would be able to realize transition to growth even during the ongoing reproduction period $\mathbb{t}$. Well, yes, in purely mathematical terms. In practical terms, however, it is more than doubtful that economic agents would be willing to *disinvest* this amount. Therefore, demand is insufficient for the existing supply, and the reallocation of producer goods for the purpose of economic growth fails.

The lack of demand we found obviously does not correspond to the view of lack of demand held by classical demand theory - Sismondi, Malthus, and later Keynes - according to which the desire to supply develops ahead of the desire to consume, which then results in a misalignment of production and consumption. In the type of lack of demand that we are discussing, there is *no* available net income. And neither does this lack of demand have anything to do with money because we did not include money in our analysis yet. It follows that Say's Law is not only wrong in the sense of Walras, as has always been claimed by demand economists; it is also wrong in the sense of Lange[3], i.e. for

[3] **Oskar Lange**, "*Say's law: a restatement and criticism*", in Oskar Lange, *Studies in Mathematical Economics and Econometrics*, 1942, pp. 49-68.

a hypothetical economy without money (a non-money economy). We conclude that a lack of demand in practice does not only arise when - and especially not when - money has disappeared somewhere, but because the laissez-faire market economy, every now and then, demands of its agents that they disinvest their *previous,* already purposely invested savings, which amounts to consuming part of the amortization. This may be called negative saving.

A mathematician who looks at equation (d) quickly discovers how negative saving can be avoided and equilibrium saved: simply make vector $\mathbf{y}_K^t$ in term 1 bigger until the lack in demand (gap Γ) closes. This vector can be enlarged by raising the prices of producer goods. In economic statistics, this price increase is closest to the *Producer Price Index* (PPI). But before we proceed with the mathematical analysis, let's illustrate our mathematical conclusion about the price increase with our numerical example above. It shows how investments cause a macroeconomic imbalance. If the economy has a structure like the one in our example, it is Sector 1 or Sector 2 - or both - that can raise their prices accordingly to bring the economy into equilibrium. The easiest way to illustrate this is if only sector 1 increases the price of its total output by 90.



Left part of equality (**d**):

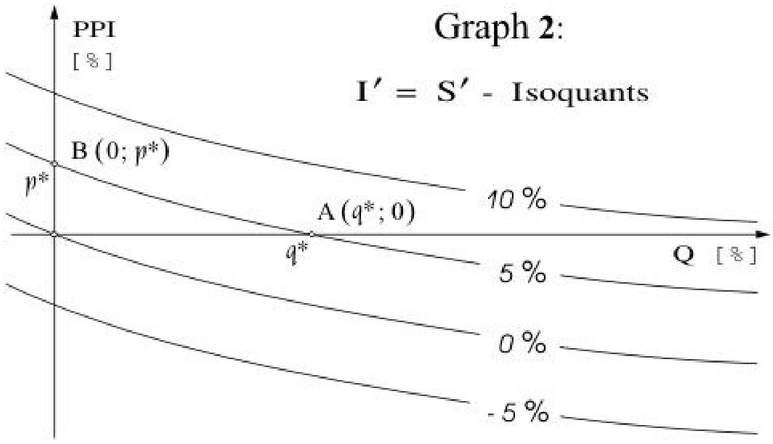
Sector 1:	1090
Sector 2:	910
Sector 3:	2000
=================	
Net income:	= **4000**

Right part of equality (**d**):

Sector 1:	0
Sector 2:	0
Sector 3:	4000
===================	
Consumer goods:	= **4000**

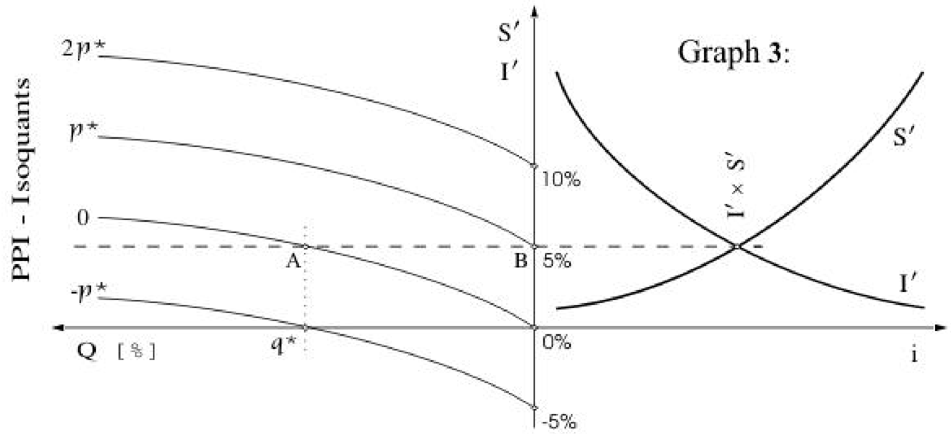
Comment: It should not be overlooked that the return of the economy to equilibrium through price changes has not changed its real structure anywhere. The price change has not changed - the value of the distributive coefficients anywhere. The coefficients δ_{12} and δ_{13} of sector 1 also have the same value - 0.443 and 0.557 - as they were before the price increase.

Aside from the raising of prices, there is another, economically important possibility to make vector $\mathbf{y}_K^t$ in term 1 bigger: raise productivity. Let's assume that some producers did increase their production in real terms in production period t, by producing more producer goods than in the previous production period from the *existing* quantity of technical resources. This productivity growth makes vector $\mathbf{y}_K^t$ in equation (d) bigger, and the gap Γ created by reallocation, smaller. If productivity growth is big enough, gap Γ disappears completely. Stronger productivity growth would eventually make reallocation obsolete. This means that price increases (PPI) and productivity growth (Q) are *complementary* variables. We represent this conclusion in a coordinate system, putting one of our variables on each axis. The unit of measurement of investment is money, but this does not prevent us from stating investment in our graphs in relative terms, i.e. in percentage terms of the total net income in the economy (-5%, 0%, 5%, 10%). Quantifying investment in this fashion makes sense because it is *financed* by net income. If no money is hoarded anywhere, the entire amount of saving (S′) is equal to the entire amount of investment (I′).



The diagram clearly demonstrates our conclusion that price increases and productivity growth are complementary variables. Thus, at a savings and investment rate of 5%, which can be realized at a productivity growth q^* (point A), or, alternatively, at a price increase p^* (point B). The practical consequences are obvious: if the economy suffers from a lack of innovation, equilibrium can be saved by reallocation at higher prices. The same is true in reverse, as shown in Graph 2 also. If there is a stronger price increase, reallocation can be larger, too. Growth can take off more strongly. Increasing prices generally have a growth-inducing effect, as will be shown later.

The previous graph shows equilibrium conditions in the economy in a plausible manner, however this type of representation is not common. We therefore make some changes to the graph, to show our results in more familiar-looking notation and diagrams. Investment and saving will be shown on the vertical axis, while the different levels of the PPI will be represented as isoquants. We also attach a familiar-looking diagram from macroeconomics to the vertical coordinate axis.



The graph is at the same time organized in a way that causality between the variables in our circular flow model is easily tracked. Price changes and productivity on the left-hand side of the graph determine the amount that may be invested or saved, and this is the level at which the I′-curve and the S′-curve must intersect on the right side of the diagram - if equilibrium is to be maintained. This means that "saving and investment are the determinates of the System, not the determinants", as Keynes already formulated. However, the *determinants* variables of the system, in our case, are not those that Keynes was referring to. He was looking for them exclusively in the psychic and monetary sectors of the economy[4], whereas in our model these variables are (for now) the price level (PPI) and productivity growth (Q). In Keynes, it was about money, or rather, about the hoarding of money (liquidity trap, real asset effect), about the lack of validity of Say's Law in the Walrasian sense; we are refuting the validity of Say's law in the sense of Lange. All of this sounds unusual and strange, but it shows the intention of our circular flow analysis, and so it is in order to say something more on it.

The conclusion from our analysis that prices and productivity determine saving and investment, and with that equilibrium and growth, is fundamentally different from what we know from our neoclassical ("neo-liberal") theory. In this theory that has become the economic mainstream in the last decade, the market is always about equilibrium, and growth is only determined by cost - above all through interest, wages and taxes. The two views are so opposed to

[4] "Saving and investment are the determinates of the System, not the determinants. They are the twin results of the system's determinants, namely, the propensity to consume, the schedule of the marginal efficiency of capital and the rate of interest." (*General Theory*, Macmillan Press, 1960, pp. 183)

each other that one cannot but use the well-known concept of paradigm by cognition theorist Thomas Kuhn. The particle-mechanic model of equilibrium (and disequilibrium) are two different paradigms or systems of thought. Both models are internally consistent, but they are not compatible. They are based on different assumptions (axioms), the main concepts and variables are different, and the relevant links between them are different, too. If you switch from one paradigm to the next, the world in the same research area "looks different and familiar objects appear in a new light", as though "you had been put onto a different planet", to speak with Thomas Kuhn. Therefore, the theoretical explanation of the functioning and of growth in the circular flow theory is different in principle, just like the explanation of the facts.

But why do we need a new paradigm? Science is sometimes forced to look for a new way of thinking, or new paradigm, if the theories generally used to explain the facts turn out to be so-called paradoxa and anomalies (Kuhn, 1970). The latter can always be brought in line with a paradigm's core statements, but this does not lead the science forward, it would degenerates (Imre Lakatos, 1977). A paradigm that is fraught with additional ad hoc assumptions is not convincing *on the one hand,* and *on the other hand* it cannot be used for prediction, which is even more important. This also goes for the neo-classical paradigm. What follows directly from the particle-mechanic reference model, stands in opposition to the facts. In regards to prices and also to productivity, reality in the neoclassical paradigm stands on its head. As these variables are particularly important in our circular flow analysis, something more needs to be said on them.

1.1 Prices, or rather, inflation as a factor in economic growth

If the price level (of producer goods) is a factor in equilibrium, this has far-reaching consequences for monetary theory. If the price level is determined by money, which cannot be doubted - even though not as strcitly as the quantity theory of money has it - money cannot be *neutral.* If an additional quantity makes prices rise, equilibrium is *eventually* possible at a higher production level. This means that at higher prices there can be a larger real reallocation, without there being disequilibrium. This is a very unusual result. Money was neutral in all the analyses and models so far - the particle-mechanic model by Walras is the best example for this - in our approach it is not neutral. This means that the circular flow model using the distribution coefficients is an increase in analytical complexity, which makes it possible to capture more of the quantitative inter-relationships. This confirms what sociologist and system analyst Niklas Luhmann stated: “Only more complexity can reduce complexity”.[5]

If prices are to have a positive impact on growth, this should also be supported empirically. We will not present statistical evidence to show this connection - it would go beyond the framework of this treatise - but limit ourselves to the general conclusions by those that researched this link. Let's start with the historians.

> "Europe experienced in the sixteenth century a continuous inflation of unprecedented proportions ... rising prices stimulated a general business expansion, ... Part of the explanation for the upsurge in prices is to be found in the influx of precious metals, especially silver, from the New World: in the second half of the sixteenth century, the international economy was in a phase of silver inflation ... that the total volume of production seems to have been insufficient to satisfy the demand. In the first half of the seventeenth century the tempo slackened. Prices began to yield ... the middle years of the seventeenth century ushered in a period of decline or stagnation that lasted for the rest of the century."[6]

Among the specialized economists Pierre Boisguillebert (1646-1714) is the first who linked high prices explicitly with a prospering economy. One would also mention David Hume who is close to these events time-wise. He proposed a strong version of the neutrality of money - he is among the inventors or developers of the quantity theory of money -, even so he admitted that

> "... it is certain that, since the discovery of the mines in America, industry has encreased in all the nations of Europe ...; and this may justly be ascribed, among oher reasons, to the encrease of gold and silver."[7]

US-economist and economic historian Walt W. Rostow examined the further development of capitalism. He concludes from his empirical research that:

> "In addition to confiscatory and taxation devices, which can operate effectively when the State is spending more productively than the taxed individuals, inflation has been important to several take-offs. In Britain of the late 1790's, the United States of the 1850's, Japan of the 1870's there is no doubt that capital formation was aided by price inflation, which shifted resources away from consumption to profits."[8]

In the fall of 2008, when something happened that according to the neo-classical (neo-liberal) mainstream should not ever have happened, it was tried to understand the “impossible” better by looking back to the economic

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[5] **Niklas Luhmann,** *Soziale Systeme*, pp. 49.
[6] **Cipolla, M. C.** (editor), *The Fontana Economic History of Europe,* Volume 2, Chapter 6, pp. 431.
[7] **Hume, D.**, *The philosophical Works of David Hume*, Vol. III, pp. 313.
[8] **Rostow, W. W.**, *The stages of economic growth*, pp. 48.

experience in between the two world wars. Germany, in fact, is a very rich field in experience. However, as far as real growth of the German economy is concerned, the inflationary years are among the best in the first third of the 20th century. If real production in the year 1919 was at only 37% of that of the prewar (1913), in the year 1922 it was already at 70%. Carl-Ludwig Holtfrerich, a German economist and economic historian who looked closely at this inflation, found that

> "In the year 1922, when development went towards hyperinflation, there was a shortage of labor, i.e. a situation of overemployment, at an unemployment rate of less than 1 %.
>
> The German economy, fuelled by inflation, acted as a ‚locomotive' for the world economy, being the only one among the large industrial countries. Inflationary policies in Germany should provide an explanation for the fact that the sharp contraction of the world economy of 1920/21 was already overcome in 1922."[9]

The reforms following the hyperinflation - such as restrictive money policy, the pressure on wages, cuts in social services, the transfer of government debt on the population, well, eventually even deflation imposed by law by Chancellor Brüning (1931) that ruined the economy and paved the way towards a faschist dictatorship. To understand Hitler as a result of inflation, fits well into neo-classical theory, but not with the facts. If today, we are to choose between the neo-liberal reforms and "Keynesian-type" inflation, we cannot say that the choice would be so difficult due to an extraordinary situation. Today's situation is not unique, it is not even rare from a historical perspective. Quite a few people that have a name in our field, such as Lester Thurow and Joseph Schumpeter, would confirm this:

> "Capitalism can live with inflation, even at a high level. Many countries, among them China, have grown rapidly at inflation rates of no less than 10 to 15 percent. However, in the last century, no capitalist society was able to grow in an environment of deflation and sinking prices. Systematic deflation always produced negative GDP growth. Once it has taken off, it is very difficult to stop."[10]
>
> "Gold or other inflations would still speed growth in the economy, deflation would hamper it."[11]

1.2 Productivity (innovations) as a factor in economic growth

The view that more productive technologies drive and reinforce growth is not a new theoretical approach. It makes the well-known theory of the *dynamic*

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[9] **Holtfrerich, C.-L.**, *Die deutsche Inflation 1914-1923*, pp 199 and 329.
[10] **Thurow, L. C.**, *Fortune Favors the Bold*, Chapter 8, original version slightly modified for the German edition (2004) by the author.
[11] **Schumpeter, J.**, *Theorie der wirtschaftlichen Entwicklung*, pp. 335.

entrepreneur by Schumpeter come to mind. In Schumpeter's opinion an expansion can only begin through innovative ideas. Our circular flow equilibrium analysis can finally resolve the issue of where Schumpeter was right and where he is wrong, and why one of the most interesting economic theories of the previous century was never able to fully assert itself.

Innovations in Schumpeter are the *modus operandi* of the economy, and they also promote growth. But it is not possible for the economy to create the necessary purchasing power of itself. The *creation of money by banks* is required. This very unorthodox view of Schumpeter's was not understood even by the traditional theory of his time. It is indeed quite problematic. For why is the additional purchasing power by the banks needed if investment goods (like all other goods) can be demanded by the material cost (amortization) that arose in their production plus the additional net income. Why is the saving by the private households and by firms not sufficient also for investment by the innovators? Here Schumpeter is obviously not respecting Says Law. He is eventually trying to save himself by a microeconomically doubtful assumption: innovators would be destitute outsiders - the proverbial puzzle freaks from the garage - that are fatefully depending on bank credit. Schumpeter therefore splits the economy into a technologically stagnating one that finances itself endogenously, and an innovative one that has to finance itself by money i.e. bank credit, that is *exogenously*. These are the real analytical problems and contradictions within the theory of the *dynamic entrepreneur*.

Our analysis shows that the economy does not require any exogenous purchasing power. It is created entirely within the system. Even in innovative investments, purchasing power that is existing somewhere is only transformed. In the previous diagram, for instance, productivity growth of, say q^* percent makes a saving and investment rate of 5% possible (point A). In Schumpeter, however, the missing purchasing power has to first be created by the banks:

> "This other method of creating money is the creation of money by the banks. No matter what shape it takes … it is never about the transformation of purchasing power that has existed before but about the creation of new purchasing power out of nowhere. …
>
> The banker is therefore not so much, and not in the first place, an intermediary dealing in the good ‚purchasing power' but she is above all the producer of this good."[12]

[12] **Schumpeter, J.**, *Theorie der wirtschaftlichen Entwicklung*, pp. 109-110.

> "Credit essentially is the creation of purchasing power for the purpose of giving it to the entrepreneur, therefore it is more than the granting of existing purchasing power."[13]

It is strange that Schumpeter makes the innovations depend on the newly created demand despite not being a demand-side economist. It is even more strange that he saw the creation of new demand in the credit issued by the banks, which was never agreed to by the most famous demand-side economist, J. M. Keynes:

> "The notion that the creation of credit by the banking system allows investment to take place to which ‚no genuine saving' corresponds can only be the result of isolating one of the consequences of the increased bank-credit to the exclusion of the others."[14]

The idea that credit is not based on real saving is short-sighted and leaves something out. But what does it leave out? Let's think about it. Even if we admit that the banks create new credit out of nowhere (*fiat money*), it is nothing, as long as it is not used for investment - a number or symbol on a piece of paper or stored away on a computer. If it is set in motion, i.e. if investment goods offered on the market are bought with it, the producer of these goods gets her income. Now we get to the question where the purchasing power implied in the income comes from: does it come from credit or from the goods? Common sense tells us that new income has to come from goods, and this is for a specific reason: If there weren't these (real) goods, there would not be any credit. A further reason is that credit ceases to exist in the moment that the transaction is completed. After this there is only the income of the seller of investment goods, and this lands on a bank account. As long as this income is not used any further, it is saving as was said by Keynes. The banker was only an intermediary, dealing in the good "purchasing power". As an aside, in monetary theory, which is known as the banking school, this is called "reverse causation". It is a mystery why deep-thinking economists such as Schumpeter did not want to know anything about the arguments of this school.

Schumpeter's statements regarding the further process of the economic cycle, are even more strange. A boom would only be possible through innovative investment, the economy would only really get started when new products "enter the market after a couple of years". A boom is therefore the consequence of expanding production "triggering massive demand by entrepreneurs, which essentially implies new purchasing power".[15] At this point in the economic cycle Schumpeter believes to have found an expansion of demand of a specific

[13] **ibid**, pp. 153.
[14] **Keynes, J. M.**, *General Theory,* pp. 82.
[15] **Schumpeter, J.**, *Theorie der wirtschaftlichen Entwicklung*, pp. 337.

kind, and his analysis finally leads into a dead end. How on earth could one of the greatest admirers of Walras, who views demand and supply as always and necessarily identical - *horribile dictu* - explain that demand all of a sudden is "massive" and the purchasing power is "substantial"? If this only means that during a boom both, demand (purchasing power), as well as supply (production) expand, then this statement is trivial. Schumpeter might be committing a logical mistake of the *circulus vitiosus* type: Production rises, because demand grows, and demand grows because production rises.

However, aside from the deficiencies in Schumpeter's views on demand, he is right in the sense that he makes innovations an important factor in the recovery of a stagnating economy. We can unconditionally agree that the creation of money by the banks substantially contributes to growth, and: they do it from the supply side since credit encourages those enterprises that have the know-how but do not have the means. They do it from the demand side because the creation of money supports prices, or even makes them rise, and therefore enables equilibrium at a higher level of production. This connection can be demonstrated mathematically only in the framework of the ciruclar flow model as we have done. We have also shown this link in our diagram: the saving and/or investment rate of 5% is possible without productivity growth when prices rise by p^* percent (point B). Schumpeter however did not mean it this way. Jürg Niehans rightly pointed out that it was Schumpeter's tragedy that he lacked a mathematically plausible model, without which intellectual thought does not form a coherent system (paradigm), and a vision is not worth much.[16]

We should also mention that everything we said about productivity growth, is also true for new products. Schumpeter was also thinking about them when he talked about innovations. New products raise profitability, such that they have exactly the same effect on equilibrium as the already discussed productivity growth. The variable Q in our mathematical analysis captures the *qualitative* change or expansion of supply (output) of any kind - even that triggered by innovations that surpass the use value of products.

2. Growth analysis based on demand theory

If reallocation happened in reproduction period t, the economy did not grow yet, assuming that productivity did not grow. The economy would only be structurally preparing for growth in reproduction period t. This must not be

[16] **Niehans, J.,** Economics: *History Doctrine*, Science, Art, in: Kyklos, Volume 34, pp. 175: " … vision is not enough. The essential step is to formalize it into an analytic model. This is what makes the idea communicable to others. … Schumpeter was a tragic figure in the history of economic analysis, because he failed to transform the vision of innovation into an analytic model."

disregarded. Our general equilibrium analysis does not include growth but only the transition towards growth. Real growth can only happen after reproduction period $t+1$. Only then the economy will be on the growth path. After this it can reproduce more producer goods (y_1, y_2, ... y_h) in each subsequent reproduction period than in the previous one. We call the sum of these goods Y_K, and its growth Y_K'. This growth makes new net investment I' possible, which can be written as

$$Y_K' \quad = \quad I' . \tag{e}$$

If we write Y_K' in matrix notation

$$Y_K' \quad = \quad \mathbf{1}\,\mathbf{\Delta}_K^{t+1}\,\mathbf{y}_K^{t+1} \; - \; \mathbf{1}\,\mathbf{\Delta}_K^{t}\,\mathbf{y}_K^{t} \tag{e'}$$

and insert it into equation (e), we obtain

$$\mathbf{1}\,\mathbf{\Delta}_K^{t+1}\,\mathbf{y}_K^{t+1} \; - \; \mathbf{1}\,\mathbf{\Delta}_K^{t}\,\mathbf{y}_K^{t} \quad = \quad I' . \tag{e''}$$

When production technology (the technical coefficients), remains the same, there is, of course, no new (net) investment. This goes for static equilibrium, as well as for the transition to growth. This is why there are no technical coefficients in equation (d). If equation (d) is to apply to a growing economy, it has to absorb equation (e''), so to speak. Both equations, however, have to be rearranged first.

Equation (d) applies to reproduction period t, however our analysis is one step ahead time-period wise, it happens in reproduction period $t+1$. We have to replace index $t+1$ by t, and t by $t+1$. If we omit the parentheses at the same time, equation (d) can be written as

$$\mathbf{1}\,\mathbf{y}_K^{t+1} \; - \; \mathbf{1}\,\mathbf{\Delta}_K^{t+1}\,\mathbf{y}_K^{t+1} \; + \; \mathbf{1}\,\hat{\mathbf{y}}_C^{t+1} \quad = \quad \mathbf{1}\,\mathbf{\Delta}_C^{t}\,\mathbf{y}_K^{t} \; + \; \mathbf{1}\,\hat{\mathbf{y}}_C^{t+1}. \tag{d'}$$

If we now take the second term in equation (e'') and put it on the right-hand side, and subsequently replace the right-hand side by the second term in equation (d'), we obtain

$$\underbrace{\mathbf{1}\,\mathbf{y}_K^{t+1} \; - \; \mathbf{1}\,\mathbf{\Delta}_K^{t}\,\mathbf{y}_K^{t}}_{\hat{Y}_K^{t+1}} \; - \; I' \; + \; \underbrace{\mathbf{1}\,\hat{\mathbf{y}}_C^{t+1}}_{\hat{Y}_C^{t+1}} \quad = \quad \underbrace{\mathbf{1}\,\mathbf{\Delta}_C^{t}\,\mathbf{y}_K^{t} \; + \; \mathbf{1}\,\hat{\mathbf{y}}_C^{t+1}}_{Y_C^{t+1}} \; . \tag{f}$$

The difference between the first two terms of the new equation (f), which we called $\hat{Y}_K^{t+1}$, amounts to the entire net income of those sectors, which produce producer goods. The fourth term, also the entire net income of those sectors which produce consumer goods, is called $\hat{Y}_C^{t+1}$. The right-hand side of the equation is the entire amount of consumer goods produced, called Y_C^{t+1}. In the next step we add the terms $\hat{Y}_K^{t+1}$ and $\hat{Y}_C^{t+1}$, which yields the sum of all the net income in the economy, and call it $\hat{Y}^{t+1}$.

Equation (f) now has an intermediate form as follows:

$$I' \quad = \quad \hat{Y}^{t+1} \; - \; Y_C^{t+1} \; .$$

The right side is now the sum of all net income remaining after the purchase of all consumer goods produced and thus available for the purchase of producer goods (raw materials, intermediate goods, machines). This part of net income is *by definition* net saving S', such that we obtain the following equation:

$$I' \quad = \quad S' \; . \tag{g}$$

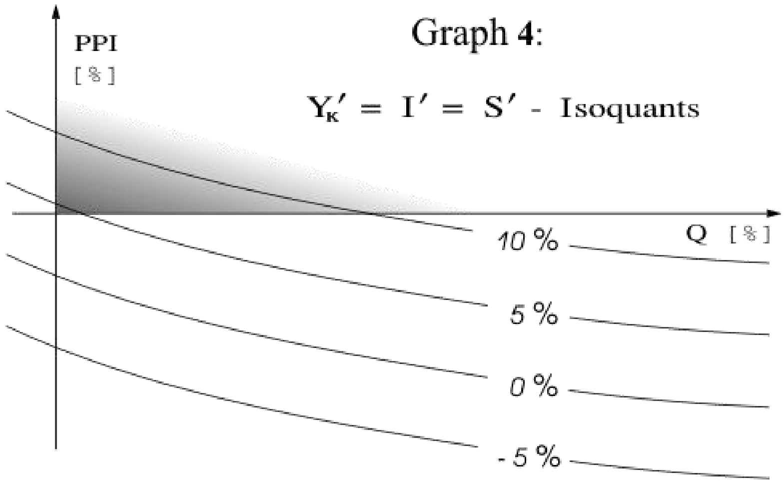
We obtain the most important result of our model which makes the paradigmatic difference from the particle-mechanic equilibrium model the plainest when we link equation (g) with equation (e), which gives us.

Zur wichtigsten Schlussfolgerung unseres Modells, die den paradigmatischen Unterschied zum herkömmlichen partikel-mechanischen Gleichgewichtsmodell am deutlichsten zum Ausdruck bringt, kommen wir, wenn wir die Gleichung (g) durch ihre Variable I' mit der Gleichung (e) verbinden, woraus folgt:

$$Y_K' \;=\; I' \;=\; S' \; .$$

This is the mathematically formulated equilibrium condition of the circular flow model, and we call it the ***general savings equation***. It shows that, in equilibrium, the variables I' and S' are quantitatively determined by Y_K'. In the framework of our circular flow analysis, it is not sufficient for general equilibrium if I' and S' are equal, they have to be equal at a certain value. Put differently, if the economy wants to invest and save more, these variables have to become bigger by the same amount. Now what determines the value of variable Y_K' in a growing economy?

If prices don't change and all sectors in the economy grow in proportion, Y_K' is exactly equal to the real growth in producer goods. This is the right-hand side in equation (e'), even if it is determined *nominally*, identical to the growth in *real* net investment I'. If in equilibrium, Y_K' is identical to I' and S', then the factors that determine those two variables, also determine Y_K'. In the analysis of reallocation we have already found that these factors are the PPI and productivity Q. We illustrated this in graph 2. If we take growth into account, the isoquants shift down, as the next picture shows. How far the isoquants shift down, depends on the further development of growth, i.e. which part of net investment I' goes to the producers of consumer goods and which part goes to the producers of producer goods.



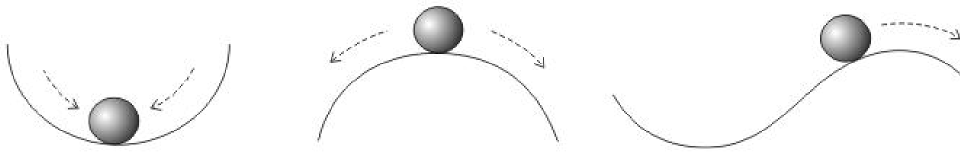
Usually, a growing economy is somewhere in the shaded area of graph 4. It follows that positive savings or positive net investments are possible in a growing economy even if price changes or productivity growth are negative, well even if *both* are negative (quadrant 3). An economy can easily grow even if the entire nominal savings volume goes to zero. Even when it becomes negative (was this not the case in the U.S. economy during the last decade?). It can also happen - in practice this is mostly the case - that real consumption grows strongly when there is too much nominal saving. Even other combinations are possible: for instance a high savings rate in an economy that is shrinking in real terms. What is true for private households, is not necessarily true for the macroeconomy, and most times it isn’t. Looking at the economy as a whole, there is no linkage between real and nominal variables. The conventional microeconomic concept of a thrifty household who attempts to increase its future consumption is useless at the macroeconomic level. The microeconomic interpretation of saving by abstinence is no more than a metaphysical item from the naïve adolescence of the economic science, when it was tried to associate something real with prices: the objective value, real abstinence, … all this amounted to - and still amounts to - looking for the “real” truth, deep down or behind the “veil” of sensual perception.

With regards to the growing economy, we can add the following for completeness. An economic boom usually drives prices up, such that the risk of saving being too high does not exist, on the contrary: In these conditions, it can even be difficult to extract enough saving from households and firms. At a sufficient level of interest this can be achieved, and firms can pay these higher interest rates during an expansion, as goods are sold easily. This explains an anomaly of neo-classical theory - and naturally of the model by Walras -, the so-called Gibson Paradox , a phenomenon that can be observed quite often: the fact that investment usually doesn't rise at lower but at higher interest rates. The correlation between investment and the level of interest, is a paradox in mainstream economic theory - in the circular flow model, it isn't.

3. A demand-based theory of crisis and the business cycle in summary

We can now easily deduct from our analysis why growth never turns into a stable equilibrium for an extended period of time, and why so many expansions collapse long before the "full-employment ceiling" is reached. If there has been an economic upturn for some time, Y_K' first loses those components that come from growth itself, because the natural and the human resources run out. The isoquants in graph 4 shift into the direction of where they are in graph 2. At the same time productivity growth Q weakens because new technical knowledge has been used up during the upturn (it materialized). Prices rise ever more slowly, or they even fall, because new productive capacity has triggered a contest of elimination. Each of these processes leads to the variable Y_K' becoming smaller and smaller. The problem for equilibrium can be easily understood from graph 3. The horizontal dashed line corresponds to the value of Y_K'. If it shifts down, the intersection $I' \times S'$ - right-hand side in graph 3 - can no longer follow at some point. If the line drops into negative, then equilibrium at the same output level may even require negative investment. As *homo oeconomicus* is not willing to disinvest, equilibrium breaks down completely, probably due to some small triggering incident. This gives the impression that the business cycle is driven by incalculable and accidental - most of the time psychological - factors. It is no wonder that people try to explain economic crashes with chaos theory. But even this attempt to explain the economic disproportionalities, has failed. Searching for disproportionalities, or "structural imbalances" as they are called, means confusing cause and effect.

When the economy crashes, not the best companies will survive, but usually those which in the critical moment happen to have the most liquidity and the lowest debt. Economic depression is thus neither a process of positive selection (Spencer's *survival of the fittest*) nor of purposeful *creative destruction* (Schumpeter). The impact of destruction does not follow any rational criterion - let alone fairness, justice, or reward for achievement. Productive resources are arbitrarily wasted and destroyed, both technical and human capital is wiped out on a large scale. It cannot even be ruled that innovators are hit the hardest, as has happened not too long ago with the *New Economy*. And only later, when a new equilibrium of economic activity has stabilized at a low level, favorable conditions, and above all new innovations, can trigger the next expansion. The laissez-faire economy resembles a constantly repeating *up and down,* as we know it from the long history of the pre-Keynesian classical market economy. We can illustrate this with a mechanical analogy, as is customary in the economic literature, arranging the circular flow model next to the model by Walras and the model by Keynes.



Neo-classical Model *Keynesian Model* *Circular Flow Model*

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